

NATIONAL AIR INTELLIGENCE CENTER



H^∞ CONTROL THEORY AND ITS APPLICATIONS IN THE AEROSPACE FIELD

by

Zhang Guohui and Do Yongfeng



Approved for public release:
distribution unlimited

DTIC QUALITY INSPECTED 3

19961024 083

HUMAN TRANSLATION

NAIC-ID(RS)T-0312-96

3 October 1996

MICROFICHE NR:

H^{oo} CONTROL THEORY AND ITS APPLICATIONS IN THE AEROSPACE FIELD

By: Zhang Guohui and Do Yongfeng

English pages: 14

Source: Cama, China Astronautics and Missilery Abstracts,
Vol. 3, Nr. 1, 1996; pp. 17-23

Country of origin: China

Translated by: Leo Kanner Associates

F33657-88-D-2188

Requester: NAIC/TASC/Richard A. Peden, Jr.

Approved for public release: distribution unlimited.

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE NATIONAL AIR INTELLIGENCE CENTER.

PREPARED BY:

TRANSLATION SERVICES
NATIONAL AIR INTELLIGENCE CENTER
WPAFB, OHIO

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

H[∞] Control Theory and Its Applications in the Aerospace Field

Zhang Guohui and Do Yongfeng

(Shaanxi Jianfeng Mechanics Research Institute)

Abstract: The basic characteristics of H[∞] control theory are described in this paper. After a brief review of robust control and H[∞] control theory, the standard H[∞] problem and its basic solution process are given. Finally its research trends in the field of aerospace are discussed.

Subject terms: *Robustness, control theory, operator

1. Theory

The H[∞] control theory, an integrating design method for control systems appeared on the horizon with the growing research on robust control systems. This is a new theory which was developed on the basis of the multivariable system frequency domain method and robustly stable anomalous value analysis method in the mid eighties. Technically, it is a systematization design method based on the optimum H[∞] index.

The "optimum H[∞] index" refers to the minimum $\|F(s)\|_{\infty}$, the H[∞] norm of the transfer function matrix $F(s)$ in H[∞] space selected in the design of control systems. The H[∞] space is a kind of Hardy space or roughly, a space where the stable transfer function is located, while the H[∞] norm of $F(s)$ is the maximum anomalous value of $F(j\omega)$ at $0 < \omega < \infty$. The H[∞] control theory can incorporate various control problems including interference suppression, robust stability, tracking control and model matching control into the standard H[∞] problem, for which a systematization integrating method is formulated.

* This is a project funded at the ministerial level which is included in the Aerospace Relay Program.

In the past decade, the H^∞ control theory has made remarkable strides in combining complex mathematic theory with practical engineering problems. This theory has attracted a huge interest because it has several outstanding advantages: first, this theory can explicitly elucidate the robust controller problem; secondly, this theory, though developed in the framework of the input and output matrix, still retains some computational superiorities of the state space method and thirdly, with this theory, designers can control the shape of the frequency response generated by the system to a considerable degree. Despite this, the H^∞ method still has a problem with its algorithm. For instance, the order of the controller is usually excessively high, which is expected to be improved further. In recent years, however, the design for H^∞ control has become easier and easier with appearance of the new version of the design tool software.

The H^∞ control theory has been widely applied in different control systems including the alternating current speed-regulating system, the inverted pendulum, and the flexible arm and robot, and its effectiveness has been increasingly accepted. Also, trial applications have spread to the aerospace field, such as space stations, flexible space structures, and aircraft attitude control.

2. Research of Robust Control and Appearance of H^∞ Control Theory

The appearance of H^∞ control theory, one of the robust control integrating methods, was inseparable from the research on robust control. It is, therefore, necessary to review over the history of the development of robust control. According to Dorato [1], the development of robust control has gone through three stages.

The first stage (before 1960) was the "classical sensitivity

design stage", during which the robustness research was based on the Nyquist frequency domain stability criterion and Black concept of large ring gain. As far as SISO system is concerned, the major concern was how to make a compromise between stability increase and sensitivity decrease.

The second stage (from 1960 through 1975) was the "state variable stage", in which heated discussion was focused on various problems concerning the state space pattern, while the robustness research was ignored, yet there were still a few research achievements in sensitivity design.

The third stage (from 1975 till now) is the "recent robust control stage". During this stage, great attention was given to research on the model uncertainty and moreover, a significant breakthrough was made in the frequency domain analysis of the multivariable system. Particularly, the transformation matrix fraction, which was used to describe the multivariable system, was introduced into the system as a design tool by Youla et al. and the Nyquist stability criterion was extended to the multivariable system [2, 3]. Also at this stage, a method of designing the multivariable system with anomalous values was constructed, i.e. the classical Bode method was extended to the multivariable system. To overcome the robustness problem confronted in optimizing the LQG design method, Doyle and Stein [4] advanced the LQG/LTR method, which was widely adopted in actual robustness designs and achieved wonderful results.

Although the multivariable frequency domain method produced a design method, the method turned out to be inconvenient in handling the robustness problem with an uncertainty, and also it was difficult to cope with the interference from an unfixed frequency spectrum. In addition, the above-mentioned robustness research achievements basically provided a measure of robustness analysis instead of a systematized integrating design method.

Nonetheless, both research achievements laid a foundation for developing a systematized robust control method. And it was based on this foundation that the H^∞ control theory finally came into being.

Back in 1981, Zames first proposed a concept of H^∞ control system design method [5]. Zames believed that the undesirable robustness in LQG design method based on the state space model was mainly caused by the integration index employed and in addition, it was unrealistic to express the uncertain interference with the white noise model. Based on this view, Zames, assuming that the interference came from a known signal set, considered using the H^∞ norm of its corresponding sensitivity function as the performance index. His design was aimed, under the worst possible interference, to reduce the system error to the minimum in the sense of H^∞ norm, so as to make the closed-loop system stable by rendering the interference problem solvable, and to feed the output of the corresponding minimum H^∞ norm index back to the controller.

In 1983, by using the Nevanlinna-Pick interpolation principle, Zames and Francis [6] arrived at an optimized solution to the optimum H^∞ sensitivity of the SISO system. In 1984, in collaboration with Helton, they presented a solution to the optimum H^∞ sensitivity of the MIMO system with the operator theory. As the optimum H^∞ sensitivity could incorporate robustness measures like the stability margin into the optimum H^∞ index, the research on the optimum mixed sensitivity was going on and achieved desirable results in applications. Of course, other famous scientists including Doyle and Stein, Safonov, Kimura and Vidyasagar also made significant contributions to the formation of the H^∞ theory.

At present, most of the existing mathematics associated with

the classical function and operator theory can be directly applied in H^∞ design and additionally, the problem of the stable compensator of the multivariable linear system, which can satisfy H^∞ norm restraints, has been thoroughly solved. Several different methods have been developed in the H^∞ theory. The first method is designed to derive a cluster solution by extending the Beurling-Lax theory to Krein space; the second method is to extend the Nevanlinna-Pick interpolation theory and adopt a series of complex value matrix algebraic operations; the third method is aimed to simplify the problem, based on the achievements made by Admjam et al. into an optimum Hankel norm approximation from stable operators to unstable operators; the fourth method is to prove, based on Riccati equation solution, that the optimum H^∞ controller can be obtained by solving two Riccati equations, with the same number of orders as the controlled target.

3. H^∞ Standard Problem

Generally speaking, Hardy space H_2 is a space where all kinds of $X(s)$ are located. Their values are picked at C^n and analyzed at $\text{Re } s > 0$, and can satisfy square Lebesgue integrable conditions:

$$\left[\sup_{\xi > 0} (2\xi)^{-1} \int_{-\xi}^{\xi} \|X(\xi + j\omega)\|^2 d\omega \right]^{1/2} < \infty \quad (1)$$

where $\|\cdot\|$ is Euclid form; sup is the determined upper boundary; H_2 form is the value on the left of Eq. (1), written as $\|X\|_2$. The physical significance of $\|X\|_2^2$ is the energy of $X(t)$ ($X(t)$ is the inverted Labesgue transformation of $X(s)$); H^2 is a Hilbert space.

Hardy space H^∞ is a space where all kinds of function $F(s)$ are located. Their values are selected at C^∞ and analyzed at

$\text{Res} > 0$, i.e.

$$\sup\{\|F(s)\|: \text{Res} > 0\} < \infty \quad (2)$$

where $\|\cdot\|$ is the spectral norm; H^∞ norm is the value on the left of the above equation; all the real-rational, proper and stable matrixes in H^∞ space formulate a sub-space of H^∞ , written as RH^∞ . For $F(s) \in RH^\infty$,

$$\|F\|_\infty = \sup\{\|F(j\omega)\|: \omega \in R\} \quad (3)$$

It can be seen from the above description that the effect of the interference with a limited power spectrum on the systematic error can be minimized ultimately as long as the H^∞ norm of the transfer function matrix from system interference to error reaches a minimum value. Study shows that several system design problems, such as interference suppression, tracking control, narrow robustness and model matching, can be incorporated into the H^∞ design problem with only one pattern, i.e. H^∞ standard problem.

Take the system shown in Fig. 1 into consideration, where w , u , z , y are all vectors, among which w is an external signal (such as a given value, an interference signal, etc.), z is controlled output (such as a tracking error, a model matching error, etc.), u is control input and y is measured output; G is a control target in a broad sense; K is a controller.

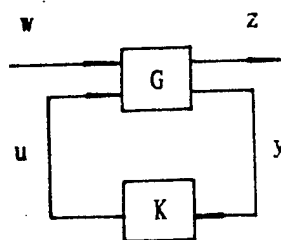


Fig. 1 Standard H^∞ Structure

Generally, G and K are selected as the real-rational transfer matrix. If G is cut into several blocks as:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (4)$$

then the following can be derived from Fig. 1:

$$z = G_{11}w + G_{12}u \quad (5)$$

$$y = G_{21}w + G_{22}u \quad (6)$$

$$z = Ky \quad (7)$$

and the transfer function matrix from w to z is

$$G_{wz} = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \quad (8)$$

The standard H^∞ problem is to find a real-rational and proper controller K and, under the condition that G is calmed by K , to ultimately minimize the H^∞ norm of the transfer matrix from w to z . This problem can be transformed to a model matching problem through using the parameterized method advanced by Youla et al.

The effectiveness of the H^∞ design method can be confirmed by the following narrow robustness problem.

Let the system be the same as in Fig. 2(a), where P is a nominal target, while P is an unknown perturbation.

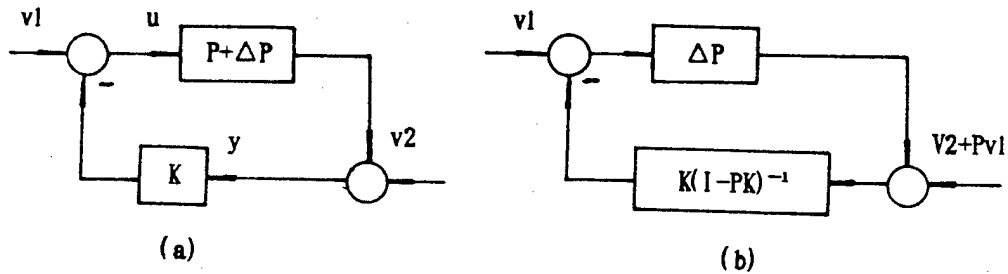


Fig. 2. Narrow Robustness Problem

This is a non-structural uncertainty. Let P and ΔP be stable, real-rational and proper, then for all $0 \leq \omega < \infty$, ΔP can satisfy

$$\|\Delta P(j\omega)\| < |R(j\omega)| \quad (9)$$

where R is a scalar boundary function in RH^∞ . K is designed the way that the nominal target is stable, i.e. $K(I-PK)^{-1} \in RH^\infty$. The narrow robustness problem is: for the designed K , what is the utmost value of $|R|$, which can still ensure a stable system?

Fig. 2(b) is derived through transformation of the block diagram. According to the principle of minimum gain, when

$$\|\Delta PK(I-PK)^{-1}\|_\infty < 1 \quad (10)$$

this system is stable (this is just the Nyquist stable criterion in the case of a single variable). It is therefore known that the system can maintain its stability if only the following is satisfied:

$$\|RK(I-PK)^{-1}\|_\infty < 1 \quad (11)$$

This is a condition sufficient for narrow robustness, i.e. the robust stability of the system can be maintained as long as the H^∞ norm of the weight transfer matrix is smaller than a particular value.

As mentioned above, several methods have been set up for the H^∞ control theory, among which the most popular method is to turn the H^∞ control problem into an interpolation problem or Nehari (Furrier sequence) extension problem, because one kind of H^∞ control problem can be re-written as:

$$\inf_{Q \in RH^\infty} \|(T_{11} - T_{12}QT_{21})(s)\|_\infty \quad (12)$$

where $T_{1j}(s)$ is stable; $T_{12}(s)$ and $T_{21}(s)$ are square and generally selected as internal matrixes, because $Q(s)$ must be stable, and the zero point of each right half plane of $T_{12}(s)$ or $T_{21}(s)$ is the zero point of $T_{12}QT_{21}$. Suppose $\{s_i: i=1, \dots, n_r\}$ is the zero point of the right half plane of T_{21} , then the presence of vector $a_i \neq 0$ makes

$$T_{21}(s_i)a_i = 0 \quad i=1, \dots, n_r \quad (13)$$

and similarly, the presence of $a_i^* \neq 0$ makes

$$a_i^* T_{12}(s_i) = 0 \quad i=n_{r+1}, \dots, n \quad (14)$$

Therefore, the interpolation condition satisfied by the closed-loop system $\rho R(s)$

$$\rho R(s) = (T_{11} - T_{12}QT_{21})(s) \quad (15)$$

turns into

$$R(s_i)a_i = T_{11}(s_i)a_i / \rho \quad i=1, \dots, n_r \quad (16)$$

or

$$a_i^* R(s_i) = a_i^* T_{11}(s_i) / \rho \quad i=n_{r+1}, \dots, n \quad (17)$$

where $\|R\|_\infty \leq 1$ and ρ are a net "gain" parameter. Thus, the above equation can ensure that $Q(s) \in RH^\infty$ and the closed-loop system is internally stable. The optimization problem is to look for

$$\inf_{R(s) \in RH^\infty} \rho = \rho_0 \quad (18)$$

to satisfy Eqs. (16) or (17). If the optimized $R(s)$ is $R_0(s)$, the optimized closed loop will be $\rho_0 R_0(s)$, and $\|\rho_0 R_0(s)\| \leq \rho_0$.

In addition, the H^∞ problem can also be written as:

$$\inf_{Q \in RH^\infty} \|T_{12} \cdot T_{11} T_{21} \cdot - Q\|_\infty$$

This is the Nehari extension problem. The Nehari principle states that the shortest distance from a given matrix R on L^∞ to H^∞ space, $\text{dist}(R, H^\infty) = \inf\{\|R - X\|_\infty, X \in H^\infty\}$, which proves that there is always a nearest H^∞ matrix X corresponding to a given L^∞ matrix R , and $\|R - X\| = \|\Gamma_R\|$, i. e.

$$\text{dist}(R, H^\infty) = \|\Gamma_R\|$$

where $\|\Gamma_R\|$ is the norm of the Hankel operator, which is equal to the square root of the maximum characteristic value of $\Gamma_R \cdot \Gamma_R^*$. Now the problem is to solve $\|\Gamma_R\|$. Once the $\|\Gamma_R\|$ value is solved, the H^∞ controller can be derived accordingly.

4. Applications in Aerospace

It was not until 1986 that sporadic reports began to appear concerning the engineering applications of the H^∞ theory due to the difficulties in realizing its software. However, with the increase of the operation speed of microprocessors, update of the logical function and the constant improvement of the theory itself, its successful applications are now released frequently and are more encouraging, the H^∞ Robust control software packages with extremely powerful functions are now available on the market.

The H^∞ control theory has been experimentally applied in many fields. Postlethwaite et al. disclosed their achievements in applying the H^∞ design method to helicopter flight control, unstable aircraft pitch control, assembly boiler control, nuclear reactor and power plant control. Limebeer et al. described the optimum H^∞ control over the synchronous turbo-generators. Backer

et al. advanced a H^∞ design method for robust control by robot mechanical hands. All these research achievements suggest that the control systems designed with the H^∞ method have proved to be superior to those designed with the conventional methods in dealing with some complicated control problems. Owing to the complexity of the aerospace control itself, the H^∞ theory displayed even more remarkable advantages in designing aerospace control systems. It is true that the H^∞ theory has very bright application prospects in this area.

In 1988, Safonov et al. presented an example of L^∞ design for the aircraft pitch control. In this paper, the authors put it as " L^∞ design" because they were handling an unstable target with a non-minimum phase. With L^∞ control theory, they made a design for the aircraft multivariable pitch control, aimed to arrive at a design index of anomalous value loop-shaping. Compared with the frequency domain weight LQG method, the L^∞ theory can more easily produce an optimized design with a wider band and a higher stability margin without the necessity of conducting an iterative adjustment for the weight term as the LQG method usually does. Also, this design example convincingly demonstrated how the "all-pass" characteristics of L^∞ allowed an accurate shaping of the closed-loop anomalous value Bode diagram to be matched with the predetermined weight function point by point.

Again in 1991, Safonov et al. further presented an H^∞ robust control integration over a large scale space structure. In this research project, with the H^∞ control theory, they arrived at a 6-state control law for a 116-state large scale flexible space structure model, proving that a very simple 4-order model can be derived by using a compensation technique for the collocated rate feedback and random cut-off model order decrease. Of course, this 4-state model can well satisfy the anomalous value

robustness criterion in the controller design, and the bandwidth of such a controller exceeds the natural frequency of all the modalities of the original 116 state model. As for the indexes such as interference suppression, bandwidth and stable robustness, these can be quantitatively described as the mixed sensitivity H^∞ control integrity problem through the weighted function. The solution of the problem is calculated through the Promatlab robust control tool box, and its H^∞ algorithm can directly control the compromise between interference suppression and robustness and calculate the H^∞ controller at one step.

In 1991, Byun et al. presented a robust H^∞ design method and its application for the space station attitude and momentum control problem. This new method included non-linear and multi-parameter variables in the state space expression of the H^∞ control theory. This technique, when applied to the multivariable space station control, arrived at an ideal robustness concerning the turning inertia uncertainty. Additionally, this method produced a controller with an extremely high bandwidth for the single input pitch control, and an excellent stable robustness in the double input rolling/off-course robust H^∞ control design. In short, this method turned out to be very suitable for the space station control.

In 1992, Rogers and Collins came up with an H^∞ controller integrity for X-29 aircraft. By using the Pro-Matlab and Matlab Robust-Control Tool Box software, they designed an H^∞ controller, which successfully overcame the unstable modality of X-29.

In 1992, Reichert advanced an H^∞ design plan for an automatic missile driverscope. The design of an automatic missile driverscope had been a challenge in the case when there were vigorous restraints over bandwidth and the system parameters varied in a wide range, particularly when it came to the ground-

to-air missiles with high mobility requirements. In that case, the conventional practice was to select a number of characteristic points for a linear model, and to respectively design, for each characteristic point, a controller which remained unchanged during linearity (LTI). Such a controller could meet the performance requirement near the characteristic point but most probably would destroy the performance and stability of the system whenever the system dynamic state varied substantially. Under such a scenario, designers had to turn to a special gain plan which, in turn, might bring in structural complexity. However, this paper, by using the H^∞/μ technique, well solved the time-varied non-linear dynamic state problem, and designed an automatic missile driverscope based on the necessary performance indexes and robustness limitations, because the design took into account a series of uncertainty factors, including aerodynamic characteristics, elastic oscillation type, helm non-linearity, gyro and accelerometer non-linearity, wind interference and noise.

All the above-mentioned research achievements indicate the high effectiveness of the H^∞ method in complex aerospace control applications. And with further updates of the H^∞ control theory, this design method will surely spread to all fields and be even more widely applied in aerospace.

5. Conclusions

Technically, the H^∞ method as a frequency domain method is close to the engineering stage. Nonetheless, as the H^∞ control theory is a new-born theory, it has its own limitations. There are some problems which are expected to be further solved. For instance, the controller has a higher order and thereby may bring about difficulties in engineering applications. Thus, research on the model simplification becomes an important topic.

Of recent interest to researchers is how to extend the H^∞ control theory to time-varied systems, non-linear systems, distributed parameter systems, discrete system and global systems. So far, of course, some achievements have already been reported in the foregoing research areas. For instance, the self-correcting H^∞ design method advanced by Grimble is a successful example. Also, the authors have been involved in the research on self-adaptive H^∞ control. In addition, it is still unclear to what degree the H^∞ indexes can reflect the quality of engineering, which will require a lot of effort in the future. However, the final results of the foregoing research will eventually open up a new path.

This paper was received on September 27, 1994.

References

- 1 Dorato, P. Robust Control. IEEE Press, 1986
- 2 Rosenbrock, H. Computer-aided Control Design. Academic Press, 1979
- 3 McFarlane, A. and Postlethwaite, I. The Generalized Nyquist Stability Criterion and Multivariable Root-loci, Int. J. C., Jan., 1977
- 4 Doyle, J. and Stein, G. Multivariable feedback design, IEEE Trans. A. C. Feb., 1981
- 5 Zames, G. Feedback and Optimal Sensitivity Model Transformations, multiplicative Seminorms and Approximate Inverses, IEEE Trans. A. C., AC-26, 1981
- 6 Zames, G. and Francis, B. A. Feedback, Minimax Sensitivity and Optimal Robustness, IEEE Trans. A. C., AC-28, 1983